

BERKOVICH SPACES, TROPICAL GEOMETRY AND MODEL THEORY

BOGOTÁ, JULY 22-28, 2017

ABSTRACTS

Courses.

Matt Baker

Linear Series on Tropical Curves

We will explore the theory of linear series on tropical curves and its applications to problems in classical algebraic geometry. Topics to be covered include the tropical Riemann-Roch theorem, metric structure of Berkovich curves and their skeleta, the specialization lemma, and tropical Brill-Noether theory.

More specifically, I'm imagining that I will cover tropical Riemann-Roch in my first lecture, Berkovich spaces (with an emphasis on the topological and metric structure of Berkovich curves) in the second lecture, the specialization lemma in my third lecture, and tropical Brill-Noether theory in my fourth lecture.

Zoé Chatzidakis

Berkovich spaces: a different approach by Hrushovski and Loeser

These talks will present in detail the construction of Hrushovski and Loeser, which allows them to retrieve and generalise some of the existing results on Berkovich spaces of quasi-projective varieties.

Mattias Jonsson

Degenerations of complex structures and Berkovich spaces

TBA

Advanced talks.**Antoine Ducros***Variation of connected components in analytic geometry.*

I will explain how Hrushovski and Loeser’s work on the homotopy type of Berkovich spaces can be used to describe the variations of the geometric connected components of the fibers of a quasi-smooth morphism

between compact analytic spaces (the motivation being the development of flattening techniques à la Raynaud-Gruson in the analytic setting – I will perhaps say also a few words about it).

John Goodrick*Tame topology for a class of definable uniform structures*

We discuss topological methods for analyzing definable sets in a framework which simultaneously generalizes both P -adically closed fields and o -minimal ordered structures.

We call a structure M “visceral” if there is a first-order definable base of a uniform topology on M in which all basic open sets are infinite and any infinite definable subset X of M has non-empty interior. We prove some tameness conditions for the topology on a structure assuming only viscosity, and assuming in addition that M has definable Skolem functions for finite sets, we obtain a cell decomposition theorem. Furthermore, under one extra topological hypothesis (“no space-filling curves”), we define a dimension on definable subsets of powers of M which is invariant under definable bijections. Our theorems are closely related to recent work of Cubides-Kovacsics, Darnière, and Leenknegt on P -minimal fields (which are examples of visceral structures), though they were proved independently.

The results presented here are joint work with Alfred Dolich.

Mattias Jonsson*Tropical and non-Archimedean limits of degenerating families of volume forms*

Kontsevich and Soibelman have given a conjectural description of the Gromov-Hausdorff limit of a maximally degenerate family of polarized Calabi-Yau manifolds in terms of the Berkovich space attached to the degeneration. Motivated by this, Mustata, Nicaise and Xu recently studied the essential skeleton of this Berkovich space, which is a natural realization of the dual complex of a minimal model of the degeneration.

I will present joint work with Sebastien Boucksom, in which we show that the volume form induced by a holomorphic form of top degree on a fiber converges, in a suitable sense, to an explicit Lebesgue type measure on the essential skeleton.

Silvain Rideau

Stably dominated groups in valued fields (joint work with Ehud Hrushovski)

In this talk, our goal will be to study groups definable in algebraically closed valued fields and, in particular those groups that have an invariant stably dominated type (equivalently the groups G whose analytification contains a G -invariant point) as they play a central role.

We will, first, show that all Abelian groups are extensions of groups internal to the residue field by groups that are unions of stably dominated groups. We will then show how stably dominated groups can be described in terms of group schemes over the valuation ring. Finally we will use those results to show that any field definable (more generally, interpretable) in an algebraically closed is either the valued fields itself or its residue field.

Felipe Rincón

Tropical Ideals

The past few years have seen a significant effort to give tropical geometry a solid algebraic foundation. In this talk I will introduce tropical ideals, which are ideals over the tropical semiring in which any bounded-degree piece is matroidal. I will discuss joint work with Diane Maclagan studying some of their main properties, and in particular showing that their underlying varieties are always finite polyhedral complexes.

Alejandro Soto

Models of abelian varieties over rank one valuation rings

Using Mumford models and toric geometry Künnemann has constructed projective regular models of abelian varieties over discrete valuation rings. One of the main features of these models is the fact that they admit a nice combinatorial description. We will show that such models can be generalized to the setting of arbitrary rank one valuation rings.

Daniele Turchetti

Semi-affinoid curves and their automorphisms

In this talk, I will present some features of automorphisms of finite order of open discs and open annuli, motivated by two arithmetic problems.

The first part concerns a question of Galois descent, that asks to classify the forms of discs and annuli under finite, tamely ramified, Galois extension. This problem is

intimately related to semi-stable models of curves, and the results I will present are joint work with Lorenzo Fantini.

In the second part, I will discuss the issues that arise in the wildly ramified case and propose a plan to overcome these difficulties, inspired by the question of lifting finite order automorphisms of curves from positive characteristic to characteristic zero.

John Welliaveetil

Finite morphisms and skeleta

In 2010, Hrushovski and Loeser used techniques from Model theory to study the homotopy types of the Berkovich analytification of algebraic varieties. They showed that the Berkovich analytification of algebraic varieties admitted strong deformation retractions onto finite simplicial complexes embedded in the varieties. In this talk, we present an application of this striking result where we study the extent to which the Berkovich analytification of an algebraic morphism deviates from being a topological fibration. We illustrate how this deviation is controlled by a finite simplicial complex embedded in the analytification of the variety.